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06MAT21

Second Semester B.E. Degree Examination, June/July 2013
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The radius of curvature in the case of a parametric curve x = x(t), y = y(t) is,

A) $\rho = \dfrac{(x'^2 + y'^2)}{x'y'' - y'x''}$ B) $\rho = \dfrac{(x^2 + y^2)^{3/2}}{x'y'' - y'x''}$ C) $\rho = \dfrac{(x'^2 + y'^2)^{3/2}}{x'y'' - y'x''}$ D) $\rho = \dfrac{(x'^2 + y'^2)^{3/2}}{x''y' - y''x'}$

ii) The value of C of the Rolle's theorem for $f(x) = x^2$ in [-1, 1] is,

A) C = 2 B) C = 0 C) C = 1 D) $C = \frac{2}{3}$

iii) Maclaurins series expansion of cosx is,

A) $1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \ldots$ B) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \ldots$

C) $1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!} + \ldots$ D) $1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$

iv) The value of C of the Lagrange's mean value theorem for $f(x) = \cos^2 x$ in $(0, \pi/2)$ is,

A) C = 1.345 B) C = 0.345 C) C = 0 D) C = 3.2

b. Find the radius of curvature for the $y^2 = \dfrac{4a^2(2a - x)}{x}$ where the curve meets the x-axis. **(04 Marks)**

c. Prove that $\dfrac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \dfrac{b-a}{\sqrt{1-b^2}}$ where a < b < 1. **(06 Marks)**

d. Obtain the Maclaurin's expansion of $\log(1 + e^x)$ as for as the fourth degree terms. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The $\lim\limits_{x \to \pi/2} (\sin x)^{\tan x}$ takes the indeterminate form,

A) $\infty \times 0$ B) $\dfrac{\infty}{\infty}$ C) 1^∞ D) ∞^0

ii) The necessary condition for finding extreme values of f(x, y),

A) $\dfrac{\partial^2 f}{\partial x^2} = 0$ B) $\dfrac{\partial^2 f}{\partial y \partial x}$ C) $\dfrac{\partial f}{\partial x} = 0, \dfrac{\partial f}{\partial y} = 0$ D) None of these

iii) $\lim\limits_{x \to 0} \dfrac{\log x}{\operatorname{cosec} x}$ equals,

A) $\frac{1}{2}$ B) 0 C) 1 D) None of these

iv) For finding the stationary value of u(x, y, z) subject to the condition ϕ(x, y, z) = C the relation is,

A) F(x, y) = 0 B) $F = u(x, y, z) + \lambda\phi(x, y, z) = C$

C) $\dfrac{\partial f}{\partial x} = 0$ D) None of these

2 b. Evaluate $\lim_{x \to 0} \frac{\sinh x - x}{\sin x - x\cos x}$. **(04 Marks)**

c. Expand $e^x \cos y$ in a Taylor's series about the point $(1, \pi/4)$ up to the 2nd degree terms. **(06 Marks)**

d. Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^1 \int_0^1 dxdy$ is,

A) 0 B) 2 C) 1 D) 2

ii) $\int_0^2 \int_1^3 \int_1^2 xy^2 z dzdydx =$

A) 26 B) 13 C) 24 D) None of these

iii) The value of Γn is,

A) $\int_{-\infty}^{\infty} e^{-x} x^{n-1} dx$ B) $\int_0^{\infty} e^{-x} x^{n-1} dx$ C) $\int_0^{\infty} e^{-x} x^n dx$ D) $\int_0^{\infty} e^{-x+1} x^n dx$

iv) The value of $\Gamma \frac{1}{2}$ is,

A) $\sqrt{\pi}$ B) π C) $\pi^2/2$ D) $\pi/\sqrt{2}$

b. Evaluate $\iint_R xydxdy$ where R is the region bounded by the co-ordinate axes and the line $x + y = 1$. **(04 Marks)**

c. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2) dzdydx$. **(06 Marks)**

d. Show that the relation between beta and gamma functions is $\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma (m+n)}$. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) Stokes theorem states that,

A) $\oint_c F \cdot d\vec{r} = \iint_s \text{div}\vec{F} dv$ B) $\oint_c F \cdot d\vec{r} = \iint_s \text{curl}\vec{F}\hat{n} ds$

C) $\oint_c F \cdot d\vec{r} = \iint_s \vec{F}\hat{n} ds$ D) $\oint_c F \cdot d\vec{r} \iiint_v \text{div}\vec{F} dv$

ii) Green's theorem in the plane is a special case of,

A) Gauss theorem B) Euler's theorem

C) Stoke's theorem D) Baye's theorem

iii) The line integral of vector $\vec{A}$ along the curve C is defined as,

A) $\int_C \vec{A} \cdot \hat{n} dr$ B) $\int_C \vec{A} \cdot \vec{F} dr$ C) $\int_C \vec{A} \cdot d\vec{r}$ D) $\iint_S \vec{F} \cdot \hat{n} ds$

iv) The spherical polar coordinates (r, θ, ϕ) given by,

A) $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

B) $x = r\cos\phi$, $y = r\sin\phi$, $z = z$

C) x, y, z

D) $x = r\sin\theta$, $y = r\cos\theta$

4 b. If $\vec{F} = xyi + yzj + zxk$, evaluate $\oint_c \vec{F}\cdot d\vec{r}$ where C is the curve represented by $x = t$, $y = t^2$, $z = t^3$, $-1 \le t \le 1$. **(04 Marks)**

c. Find the area of the asteroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ by employing Green's theorem. **(06 Marks)**

d. Evaluate $\int_C (axi + byj + czk)\cdot\hat{n}ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The particular integral of $(D^2+a^2)y = \sin ax$ is,

A) $-\frac{x}{2a}\cos ax$ B) $\frac{x}{2a}\cos ax$ C) $-\frac{ax}{2}\cos ax$ D) $\frac{ax}{2}\cos ax$

ii) The solution of the differential equation $(D^4-2D^3+D^2)y = 0$ is,

A) $y = (C_1+C_2x^2)+(C_3+C_4x)$ B) $y = (C_1+C_2x)e^x$
C) $y = (C_1+C_2x)+(C_3+C_4x)e^x$ D) $y = (C_1+C_2x)+(C_3+C_4x)$

iii) The particular solution of the differential equation $f(D)y = xV$, where V is any function of x,

A) $\left[V - \frac{f'(D)}{f(D)}\right]\frac{x}{f(D)}$ B) $\left[x - \frac{f(D)}{f'(D)}\right]\frac{V}{f(D)}$ C) $\left[x - \frac{f'(D)}{V}\right]f(D)$ D) $\left[x - \frac{f'(D)}{f(D)}\right]\frac{V}{f(D)}$

iv) By the method of undetermined co-efficient for the linear differential equation with $(D^2+1)y = \sin x$, we assume the particular integral as,

A) $y = (A\cos x + B\sin x)$ B) $y = x(A\cos x + B\sin x)$
C) $y = (A\cos x + B\sin x)e^x$ D) None of these

b. Solve $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = 5e^{-2t}$. **(04 Marks)**

c. Solve $y'' + 4y = x^2 + e^{-x}$ by the method of undetermined co-efficient. **(06 Marks)**

d. Solve $(D^2 - 2D + 5)y = e^{2x}\sin x$. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) In the method of variation of parameters Wronskian of $\frac{d^2y}{dx^2} + y = \tan x$ is,

A) W = 2 B) W = 1 C) W = 0 D) W = 4

ii) To transform $(ax+b)^2y'' + (ax+b)y' + y = \phi(x)$ into a linear differential with constant co-efficient put t =

A) e^x B) x C) $\log(ax+b)$ D) e^{2x}

iii) The equation $a_0x^2y'' + a_1xy' + a_2y = \phi(x)$ is called,

A) Legendre's linear equation B) Homogeneous equation
C) Cauchy's homogeneous equation D) None of these

iv) The solution of the differential equation, $y'' + 4y' + 4y = 0$ is,

A) $(1-x)e^{2(1-x)}$ B) $(C_1 + C_2x)e^{-2x}$ C) C_1e^{-2x} D) $C_1e^{2x} + C_2e^{-2x}$

b. Using the method of variation of parameters solve, $\frac{d^2y}{dx^2} + y = \sec x\tan x$. **(04 Marks)**

c. Solve $x^2y'' + xy' + 9y = 3x^2 + \sin(3\log x)$. **(06 Marks)**

d. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x}$ subject to the conditions $y(0) = 1 = y'(0)$. **(06 Marks)**

7 a. Choose correct answers for the following : **(04 Marks)**

i) Laplace transform of coshat is,

A) $\frac{s}{s^2+a^2}$ B) $\frac{s}{s^2-a^2}$ C) $\frac{a}{s^2+a^2}$ D) $\frac{a}{s^2-a^2}$

ii) If $\alpha[f(t)]=\bar{f}(s)$ then $L\left[\frac{f(t)}{t}\right]$ is equal to,

A) $\int_{-\infty}^{\infty}\bar{f}(s)ds$ B) $\int_{s}^{\infty}f(t)dt$ C) $\int_{s}^{\infty}\bar{f}(s)ds$ D) $\int_{s}^{\infty}\frac{\bar{f}(s)}{s}ds$

iii) The Laplace transform of $t\cos at$ is,

A) $\frac{s-a}{(s^2+a^2)^2}$ B) $\frac{s^2-a^2}{(s^2+a^2)}$ C) $\frac{s^2+a^2}{(s^2+a^2)^2}$ D) $\frac{s^2-a^2}{(s^2+a^2)}$

iv) The value of $L[t^4\delta(t-3)]$,

A) $9e^{-s}$ B) $81e^{-3s}$ C) $12e^{-3s}$ D) $3e^{-2s}$

b. Find the Laplace transform of, $e^{-2t}(2\cos 5t-\sin 5t)$. **(04 Marks)**

c. If $L[f(t)]=\bar{f}(s)$, then $L[t^n f(t)]=(-1)^n\frac{d^n}{ds^n}[\bar{f}(s)]$ where n is a positive integer. **(06 Marks)**

d. A periodic function of period $\frac{2\pi}{\omega}$ is defined by $f(t)=\begin{cases}E\sin\omega t, & 0\le t\le \pi/\omega\\ 0, & \pi/\omega\le t\le 2\pi/\omega\end{cases}$ where E and ω are constants. Find $L[f(t)]$. **(06 Marks)**

8 a. Choose correct answers for the following : **(04 Marks)**

i) The value of $L^{-1}\left[\frac{1}{s^2-36}\right]$ is,

A) $6\sinh 6t$ B) $\frac{1}{6}\sin 6t$ C) $\frac{1}{6}\sinh 6t$ D) $\sinh 6t$

ii) The convolution of two functions f(t) and g(t) is defined in the form of integral as,

A) $f(t)*g(t)=\int_0^{\infty}f(u)g(t-u)du$ B) $f(t)*g(t)=\int_{-\infty}^{\infty}f(u)g(t-u)du$

C) $f(t)*g(t)=\int_0^{t}f(u)g(u)du$ D) $f(t)*g(t)=\int_0^{t}f(u)g(t-u)du$

iii) $L[y''(t)]$ is equal to,

A) $SL[y(t)]-y(0)$ B) $S^2L[y(t)]-y(0)-y'(0)$

C) $S^2L[y(t)]-Sy(0)-y'(0)$ D) $S^2L[y(t)]-Sy'(0)-y(0)$

iv) The value of $L^{-1}\left[\frac{1}{S^{n+1}}\right]$ is,

A) $\frac{t^n}{\Gamma n}$ B) $\frac{t^{n+1}}{n!}$ C) $\frac{t^n}{(n-1)!}$ D) $\frac{t^n}{n!}$

b. Find the inverse Laplace transform of $\frac{s+5}{s^2-6s+13}$. **(04 Marks)**

c. Using convolution theorem, obtain the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$. **(06 Marks)**

d. Solve by using Laplace transforms $\frac{d^2y}{dt^2}+y=0$, given that $y(0)=2,\ y'(0)=0$. **(06 Marks)**

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